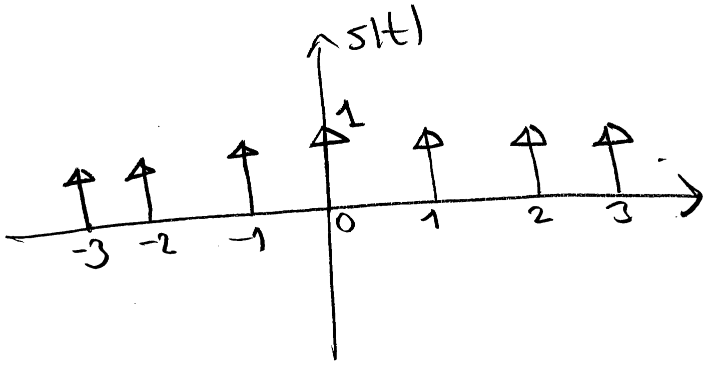
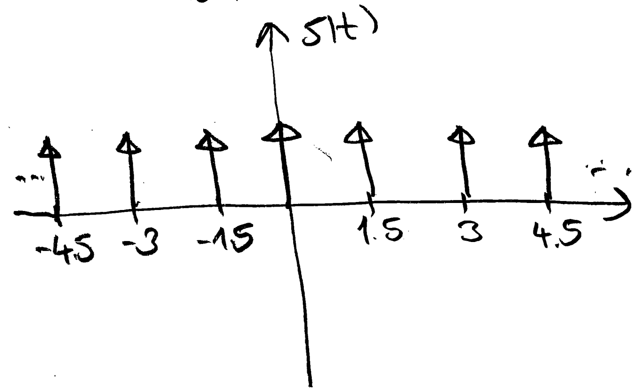
# ECE 310 HW-1 Solutions

1) $s(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$

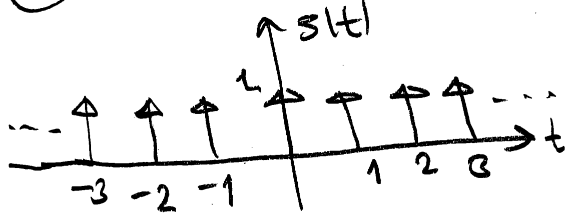
$T_s = 1$

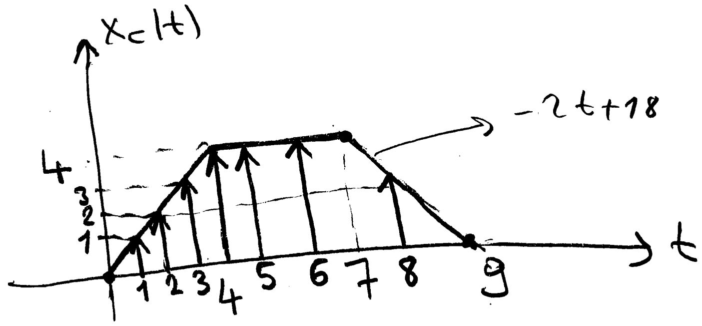


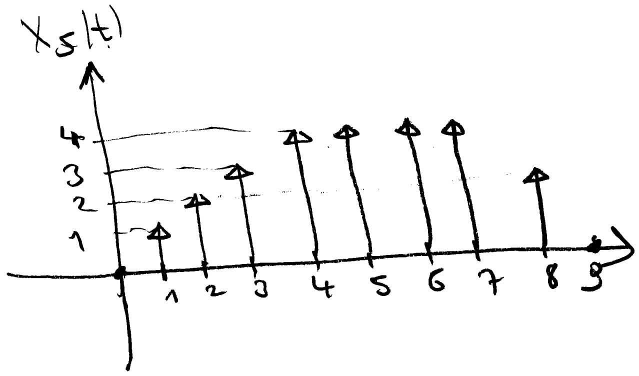

$T_s = 1.5$



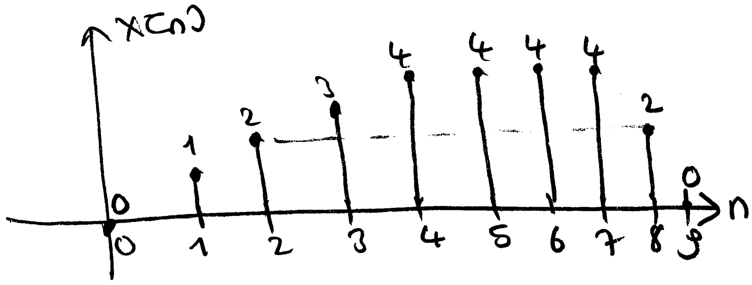

2) a) $T_s = 1\,sec$



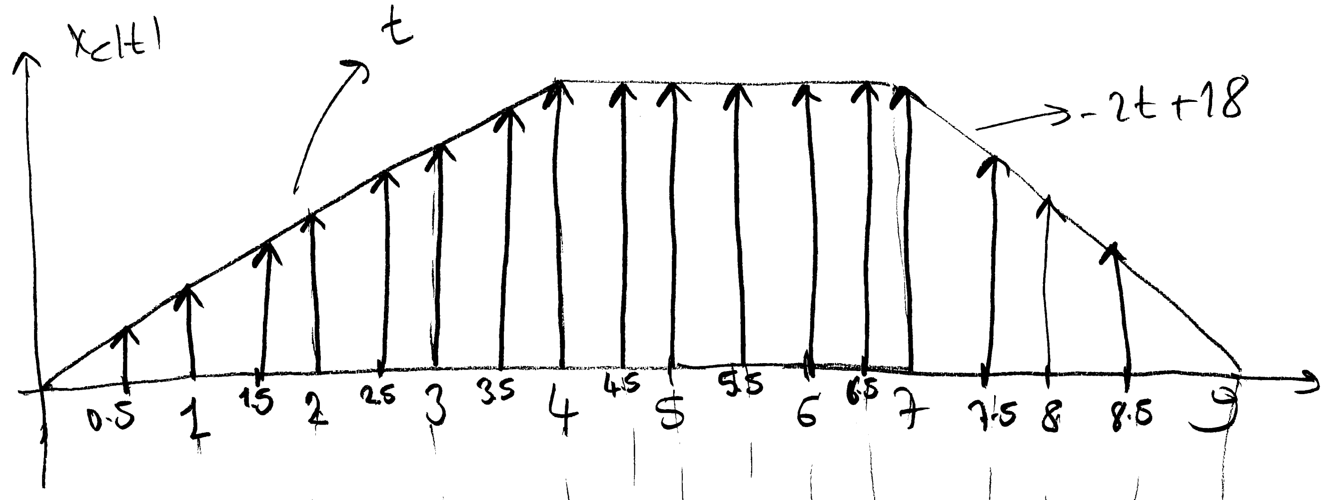



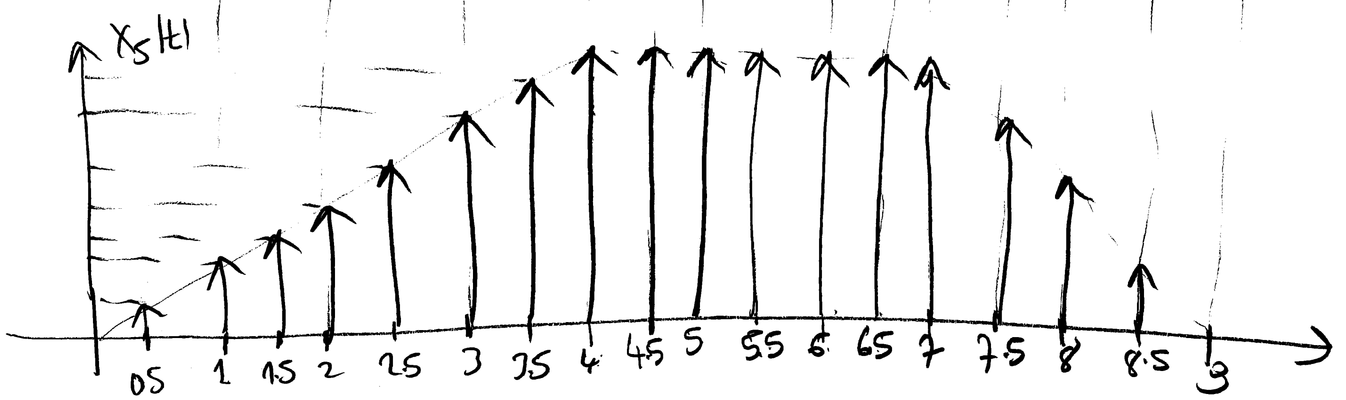




$x[n] = [\underset{\substack{\downarrow \\ n=0}}{0}\ 1\ 2\ 3\ 4\ 4\ 4\ 4\ 2\ \boxed{0}]$



② (b)




$$x[n] = [\underset{\substack{\uparrow \\ n=0}}{0} \ 0.5 \ 1 \ 1.5 \ 2 \ 2.5 \ 3 \ 3.5 \ 4 \ 4 \ 4 \ 4 \ 4 \ 4 \ 4 \ 3 \ 2 \ 1 \ 0]$$

③

$$s(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$$

$$s(t) = \sum_{k=\infty}^{\infty} S[k] e^{jk\frac{2\pi}{T_s}t}$$

$$S[k] = \frac{1}{T_s} \int_{T_s} s(t) e^{-j\frac{2\pi}{T_s}t} dt$$

$$= \frac{1}{T_s} \int_{T_s} \delta(t) e^{0} dt$$

$$= 1/T_s$$

$$s(t) = \frac{1}{T_s} \sum_{k=\infty}^{\infty} e^{jk\frac{2\pi}{T_s}t}$$

We know that $e^{j\omega_0 t} \overset{FT}{\longleftrightarrow} 2\pi\delta(\omega-\omega_0)$

$$s(t) = \frac{1}{T_s} \sum_{k=-\infty}^{\infty} e^{jk\frac{2\pi}{T_s}t}$$

$$S(\omega) = \frac{1}{T_s} \sum_{k=-\infty}^{\infty} 2\pi\delta\left(\omega - k\frac{2\pi}{T_s}\right)$$

$$S(\omega) = \frac{2\pi}{T_s} \sum_{k=-\infty}^{\infty} \delta\left(\omega - k\frac{2\pi}{T_s}\right)$$

(4)

$$x_s(t) = x_c(t)\, s(t)$$

$$x_s(\omega) = \frac{1}{2\pi} x_c(\omega) \otimes S(\omega)$$

$$= \frac{1}{2\pi} x_c(\omega) \otimes \frac{2\pi}{T_s} \sum_{k=-\infty} \delta\left(\omega - k\frac{2\pi}{T_s}\right)$$

$$X_s(\omega) = \frac{1}{T_s} \sum_{k=-\infty}^{\infty} X_c\left(\omega - k\frac{2\pi}{T_s}\right)$$

(5)

$$x[n] = x_c(nT_s)$$

$$x_s(t) = x_c(t)\, s(t) \rightarrow x_s(t) = x_c(t) \sum \delta(t - nT_s)$$

$$x_s(t) = \sum x_c(nT_s)\, \delta(t - nT_s)$$

$$X_s(\omega) = \int x_s(t)\, e^{-j\omega t}\, dt$$

$$= \sum x_c(nT_s)\, e^{-j\omega nT_s}$$

$$=$$

→ compare

$$x[n] \xrightarrow{FT} X_n(\omega) = \sum \underset{x_c(nT_s)}{\underbrace{x[n]}}\, e^{-j\omega n}$$

$$\rightarrow \quad X_n(\omega T_s) = X_s(\omega)$$

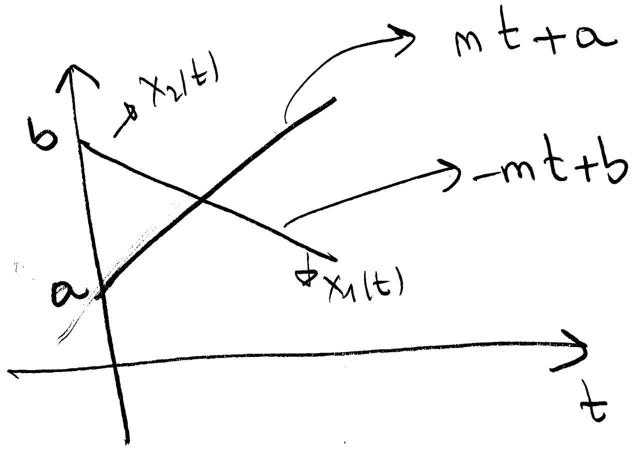
$$\rightarrow X_n(\omega) = X_s\left(\frac{\omega}{T_s}\right)$$

$$X_n(\omega) = X_s\left(\frac{\omega}{T_s}\right)$$

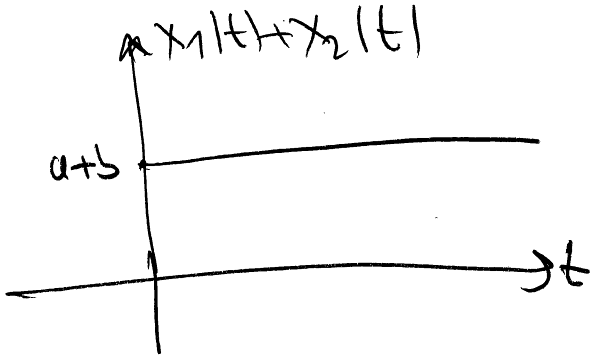
and also from the previous question we found $X_s(\omega) = \frac{1}{T_s} \sum X_c\left(\omega - k\frac{2\pi}{T_s}\right)$ put into

$$X_n(\omega) = \frac{1}{T_s} \sum X_c\left(\frac{\omega}{T_s} - k\frac{2\pi}{T_s}\right)$$

6

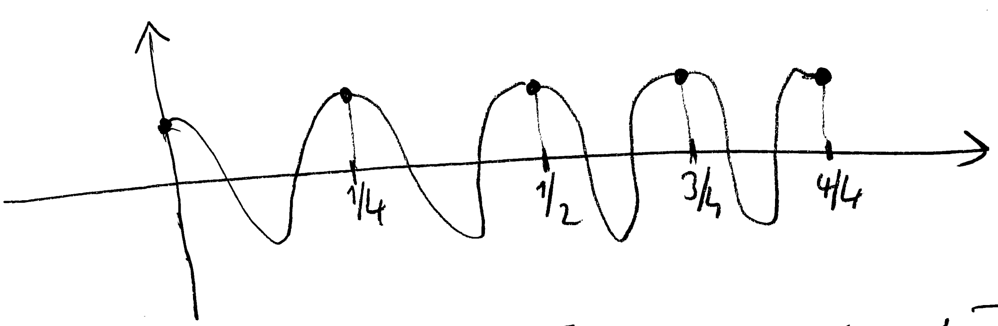



$(mt+a) + (-mt+b) = a+b$




7 $\cos(8\pi t) = \cos(2\pi f t) \rightarrow f=4 \quad T = 1/4$ signal period.

a)




$x[n] = [1 \ 1 \ 1 \ 1 \ 1]$

or $\quad x[n] = x_c(nT_s) \rightarrow x[n] = \cos(8\pi . n \frac{1}{4})$

$= \cos(2\pi n)$

$= \cos(0) \rightarrow 1$

7-b) $x[n] = x_c(nT_s)$

$= \cos\left(8\pi . n . \frac{1}{16}\right)$

$= \cos\left(\frac{\pi n}{2}\right)$

for $0 \leq t \leq 1$ $\quad t = nT_s \rightarrow n = t/T_s$

$\rightarrow n = 0 : 16$

$x[n] = \left[\cos(0) \quad \cos\left(\frac{\pi}{2}\right) \quad \cos\left(\frac{2\pi}{2}\right) \dots \cos\left(\frac{16\pi}{2}\right)\right]$

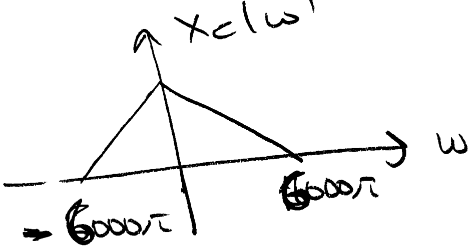
$x[n] = [1 \quad 0 \quad -1 \quad 1 \quad 0 \quad -1 \dots 1]$

7-c) in b $f_s = 16 > 2.4$

freq of cosine signal

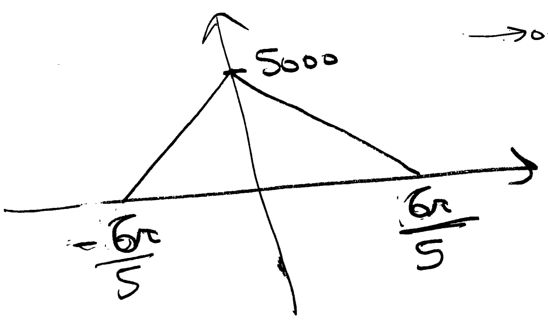
b is better

(8)

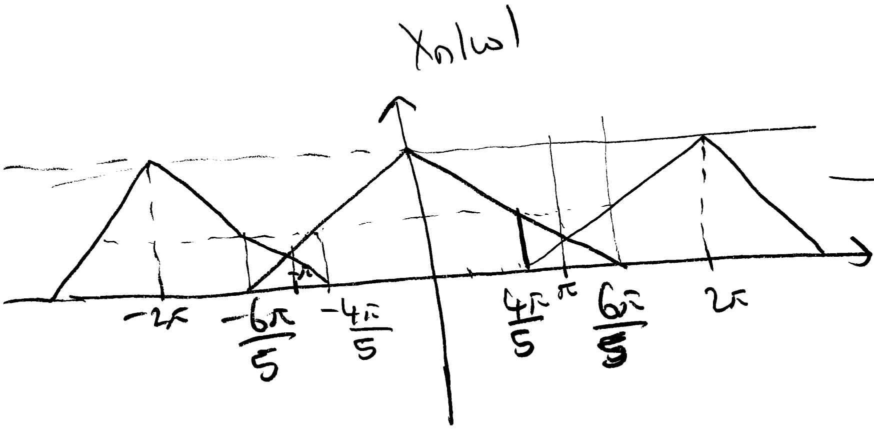



$X_n(\omega) = \frac{1}{T_s} X_c\left(\frac{\omega}{T_s}\right)$

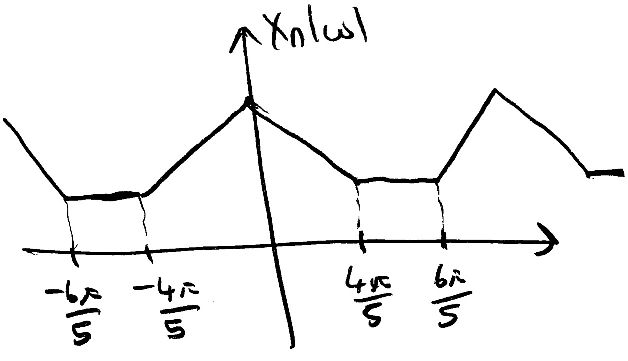
→ one period




$X_n(\omega) = \frac{1}{T_s} \sum X_c\left(\frac{\omega}{T_s} - k\frac{2\pi}{T_s}\right)$




→

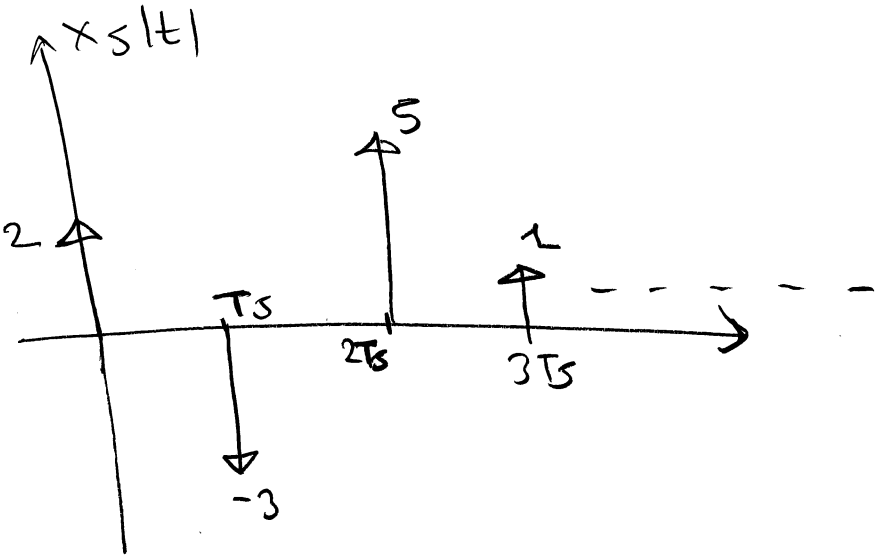


(9) $\overset{\uparrow n=0}{}$

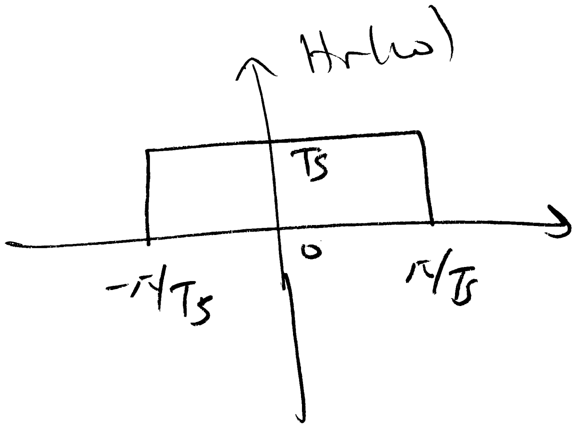
$x[n] = [2\ -3\ 5\ 1\ 2\ 3\ 1.5\ 4.3\ 2.5\ -2.5\ 2]$

$x_s(t) = \sum x(n)\,\delta(t-nT_s)$

$x_s(t) = x[0]\,\delta(t-0T_s) + x[1]\,\delta(t-T_s)$
$+ x[2]\,\delta(t-3T_s) + \cdots$




(10)




$$h_r(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} H_r(\omega)\, e^{j\omega t}\, d\omega$$

$$= \frac{1}{2\pi}\int_{-\pi/T_s}^{\pi/T_s} T_s\, e^{j\omega t}\, d\omega$$

(11) $x_s(t) = x_c(t)\, s(t) \rightarrow x_s(t) = \sum x[n]\,\delta(t-nT_s)$

$x_r(t) = x_s(t) * h_r(t) \longrightarrow x_r(t) = \sum x[n]\, h_r(t-nT_s)$